

Original scientific paper

Quantifying the Sustainable Urban Economy Through Integrated Metrics, Models, and Sensitivity Analysis

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ARTICLE INFO:

Article History:

Received: 26 June 2026
Revised 1: 15 August 2026
Revised 2: 17 September 2026
Accepted: 20 September 2026
Available online: 29 September 2026

Keywords:

sustainable urban economy;
urban sustainability assessment;
eco-efficiency; spatial equity;
composite index;
sensitivity analysis

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ABSTRACT

Urban sustainability assessment requires metrics that jointly represent ecological limits, economic performance, and distributive equity. This structured, "method-centered" review evaluates eight established quantitative approaches, including ecological footprint, composite sustainability indices, location quotient, green GDP, data envelopment analysis, Shannon diversity, Gini inequality, and net present value and then develops a Sustainable Urban Economy Integrated Index (SUEI). Twenty-five verified peer-reviewed studies published from 2021 to 2026, together with foundational sources, were coded by method, scale, dimension, data requirements, and analytical contribution. An author-developed capability rubric compares the methods across eight criteria, and four fully disclosed weighting scenarios test rank sensitivity. Because the rubric records author appraisals of analytical capability rather than observed city outcomes, all derived statistics are interpreted descriptively. Composite indices and Gini assessment tie for the highest equal-weight capability mean (4.13/5), yet method rankings vary materially across normative priorities (tie-corrected Kendall's $W = 0.525$). The revised SUEI uses directionally consistent, benchmark-based normalization, separate ecological, economic, and social subindices, geometric aggregation, and explicit safeguards against indicator duplication. Illustrative cases from Freiburg, Curitiba, Singapore, and Copenhagen clarify scale and interpretation without being treated as comparable empirical observations; their analytical units differ by roughly three orders of magnitude, which is itself evidence of why boundary choice governs interpretation. The synthesis shows that no single method is sufficient: credible assessment requires triangulating physical pressure, efficiency or productivity, equity, and, where relevant, investment evidence. The framework provides a transparent basis for future harmonized city-panel applications, sensitivity testing, and spatial validation.

JOURNAL OF CONTEMPORARY URBAN AFFAIRS (2026), 10(2), 296-318.

<https://doi.org/10.25034/jcua.2026.v10n2-1>

www.jcua.com

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Highlights:

- Eight methods are compared across ecological, economic, and equity needs
- A new index combines environmental pressure, economic strength, and equity
- Ecological priorities can sharply change which methods rank highest
- City boundaries can strongly change urban sustainability comparisons

Contribution to the field statement:

This study brings eight established urban sustainability methods into one transparent framework and proposes an integrated index linking ecological pressure, economic performance, and social equity. It shows that method rankings change with policy priorities, demonstrating why cities should combine complementary measures and report sensitivity rather than rely on one score.

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How to cite this article? (APA Style)

Athari, S. A., Azoury, N., & Kirikkaleli, D. (2026). Quantifying the sustainable urban economy through integrated metrics, models, and sensitivity analysis. *Journal of Contemporary Urban Affairs*, 10(2), 296-318. <https://doi.org/10.25034/jcua.2026.v10n2-1>

1. Introduction

Cities concentrate population, infrastructure, investment, and material throughput, making the urban scale central to how economic development interacts with ecological limits. Urban sustainability therefore cannot be reduced to environmental performance alone: it concerns whether urban systems maintain productivity and human welfare while respecting resource constraints and distributing

benefits and burdens fairly (Beatley, 2000; Kovács et al., 2022; Saraswat et al., 2025; Wackernagel & Rees, 1996). This study defines the sustainable urban economy as an urban system that sustains economic value creation without transferring unacceptable ecological costs across territories, generations and social groups. Operationalizing that definition is difficult because ecological pressure, economic output, productive efficiency, accessibility, and inequality are measured in different units and at different spatial scales. Rather than relying on a single headline measure, contemporary assessment frameworks consequently combine indicator systems, multivariate reduction, multi-criteria decision analysis, frontier-efficiency models, and spatial analysis (Foroozesh et al., 2022; Furlan et al., 2025; Liu, Yang, et al., 2022; Malah & Bahi, 2022; Marjanović et al., 2026). The methodological question is not simply which indicator is available, but what construct it represents, what it omits, and whether its assumptions remain stable across cities and time.

Quantification is essential for three main reasons. First, explicit formulas permit cross-city and longitudinal comparison under stated normalization and boundary rules. Second, separate ecological, economic, and distributional measures reveal trade-offs that an aggregate narrative may conceal (for example, declining emissions intensity can coexist with a growing absolute footprint). Third, transparent diagnostics allow researchers to test whether results are robust to indicator selection, weighting, spatial aggregation, and uncertainty. Recent work using exploratory and confirmatory factor analysis (EFA and CFA), principal component analysis (PCA), geographic information systems (GIS), the analytic hierarchy process (AHP), the best–worst method (BWM), the technique for order of preference by similarity to ideal solution (TOPSIS), machine learning, and benefit-of-the-doubt (BoD) weighting demonstrates the increasing methodological sophistication of urban sustainability assessment (Foroozesh et al., 2022; Furlan et al., 2025; Khodakarami et al., 2023; Liu, Yang, et al., 2022; Malah & Bahi, 2022; Marjanović et al., 2026).

Efficiency-oriented research adds a production perspective by treating energy, land, labor, and capital as inputs; economic value and well-being as desirable outputs; and pollution or carbon emissions as undesirable outputs. Super-efficiency slack-based measure (Super-SBM) data envelopment analysis, green total-factor productivity (GTFP), Global Malmquist–Luenberger (GML) indices, and quasi-experimental or spatial models have been applied to urban eco-efficiency, city-size effects, smart-city policies, low-carbon finance, and green innovation (Li & Cheng, 2022; Long, 2021; Wang & Jiang, 2025; Wang et al., 2022; Yao et al., 2022; Zhao et al., 2022; Zou et al., 2024). The above-mentioned studies show that sustainable economic performance depends on both the level of output and the environmental burden associated with producing it. Distribution and spatial structure are equally consequential. Green-space quantity can improve while access remains unequal, and citywide averages can obscure large neighborhood disparities (Guan et al., 2023; Lu et al., 2024; Wu & Kim, 2021). Land-use diversity, accessibility, and spatial econometric analysis further show that sustainability outcomes depend on urban form, analytical grain, and spillovers across administrative boundaries (Iannillo & Fasolino, 2021; Jiang et al., 2022; Khodakarami et al., 2023). Therefore, aggregate provision should not be interpreted as evidence of distributive fairness.

However, the main research gap is not a lack of metrics but the lack of a common, transparent framework which would allow comparing what these approaches measure, how they rank according to different normative preferences, and how they can be integrated without counting the same energy, land, or emission footprint more than once. The ecological footprint measures pressure against biocapacity (Kovács et al., 2022; Wu & Bai, 2022), green GDP monetizes depletion and environmental harm, but is entirely driven by the assumptions of the model (Zhu et al., 2023), data envelopment analysis focuses on the evaluation of relative efficiency and not absolute sustainability (Liu, Qu, et al., 2022; Long, 2021), while project evaluation assesses investment profitability, not city-wide sustainability (Wilbers et al., 2022).

This review adds value by systematically outlining eight commonly used quantitative methods, synthesizing method-focused literature up to 30 July 2026, introducing a capability matrix and weight allocation examples, and suggesting an indicator-based, benchmark-oriented Sustainable Urban Economy Integrated Index (SUEI). The SUEI includes three different pillars of ecology, economy, and social equity, restricts inter-pillar compensation by geometric aggregation and includes criteria for directionality, benchmark consistency, and overlap avoidance of indicators. The four urban cases remain as illustrative applications to better understand the scales and methods chosen and are not considered a statistically representative sample, nor are city scores calculated. Therefore, four related questions are answered in the article. Firstly, which aspects of ecology, economy, geography, distribution, and investment are captured or missed by the most popular quantitative methods? Secondly, what is the sensitivity of the methods' ranking to ecology, economy and equity concerns? Thirdly, what are the most suitable combinations for triangulation and avoiding redundant indicators? Finally, how can an integrated approach be validated in future city-panel studies? These questions structure the remainder of the article: the conceptual framework, review procedure, comparative method analysis, illustrative cases, capability and sensitivity analysis, and discussion of findings.

2. Conceptual Framework

The sustainable urban economy is organized here around three analytically distinct but interacting pillars, each associated with a different quantitative tradition:

- a) Ecological pillar: resource throughput, land and biocapacity consumption, and emissions, measured through material-flow and footprint accounting.
- b) Economic pillar: productivity, sectoral specialization, and efficiency, measured through GDP-based, location-quotient, and frontier-efficiency methods.
- c) Social pillar: distribution of costs and benefits, including income, access, and exposure to environmental burden, measured through inequality and accessibility indices.

An assessment can only be credible if attention is paid to all three pillars at the same time. The examples used in Section 5 were chosen specifically to highlight the variations in terms of the spatial scale, the boundaries for the accounting process, and the policy function.

This structure is consistent with multidimensional assessment frameworks that combine environmental conditions, socioeconomic performance, ecosystem services, infrastructure, governance, and urban form (Foroozesh et al., 2022; Khodakarami et al., 2023; Liu, Yang, et al., 2022; Saraswat et al., 2025). Since additive indices have the potential to let outstanding performance in one pillar obscure a critical weakness in another, the suggested model preserves pillar-based results and applies partial compensation within the comprehensive index. Figure 1 below illustrates this structure by tying the three pillars to corresponding physical pressure, efficiency integration, and equity and investment appraisal information. When reading from the top to the bottom, one sees the analytical path applied throughout the entire paper: the pillars tell what needs to be measured, the integrated layer tells how this information should be interpreted collectively, and the lower row lists the corresponding methods formalized in Section 4.

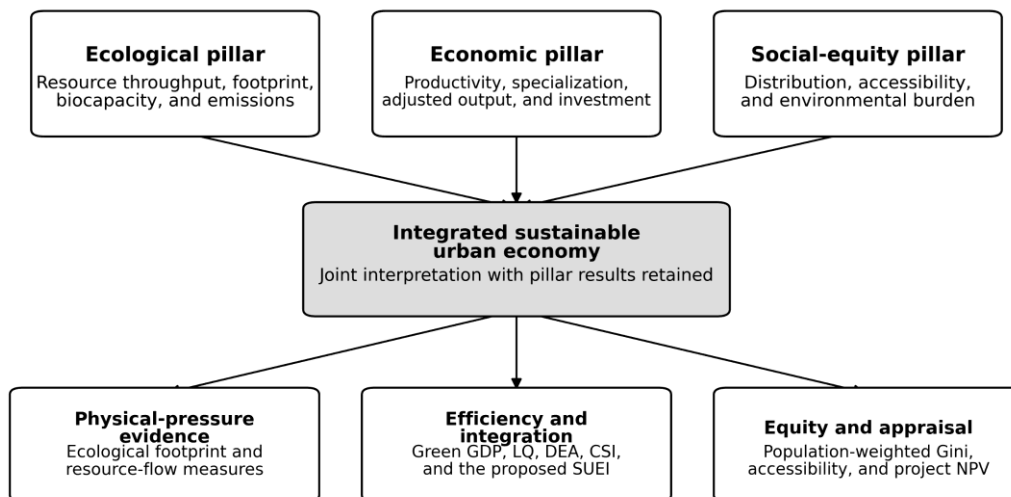


Figure 1. Conceptual architecture for quantifying the sustainable urban economy.

2.1 Triangulation Principle

The triangulation principle draws on three sets of evidence: physical pressure (ecological footprint, biocapacity, and emissions); economic efficiency and monetary evidence (green GDP, DEA, LQ, and NPV at the project level); and spatial distribution (population-weighted Gini coefficient and normalized land-use diversity). The goal is not to average these incompatible constructs indiscriminately but to test whether their conclusions converge. Divergence is retained as evidence of conflict, and Section 6 measures its extent.

3. Review Design and Evidence-Synthesis Protocol

This study employs a structured, method-centered evidence synthesis rather than an exhaustive systematic review or statistical meta-analysis. The unit of synthesis is the quantitative method and its demonstrated urban application. Outcome definitions, units, scales, and research designs differ too substantially across footprint, efficiency, composite index, equity, land-use, and investment studies to justify pooled effect sizes. The review therefore prioritizes transparent eligibility, source verification, evidence coding, capability appraisal, and sensitivity analysis.

3.1 Scope, Search Logic, and Eligibility

The evidence update was completed on 15 June 2026 through targeted publisher, DOI, and bibliographic record searches. Search combinations included “urban sustainability,” “sustainable urban economy,” “ecological footprint,” “biocapacity,” “eco-efficiency,” “Super-SBM,” “green total factor productivity,” “Gini,” “land use mix,” “green GDP,” “spatial econometric,” “composite index,” and “cost-benefit” or “net present value.” Eligible studies were peer-reviewed journal articles published from 2021 to 2026 that directly operationalized at least one focal method in an urban or city-comparable context. Foundational books and articles were retained only to define established concepts and models, and are therefore excluded from the contemporary corpus count. The final contemporary corpus contains 25 verified journal studies, supplemented by foundational works on ecological footprint, green urbanism, and DEA. A four-part quality gate required a publisher or DOI record matching the stated authors, title, year, and outlet; explicit quantitative operationalization; a defined spatial unit or analytical scale; and sufficient methodological detail to identify inputs, indicators, outputs, or valuation assumptions. This focused corpus supports method comparison but is not represented as an exhaustive census of all sustainable-city publications.

3.2 Evidence Coding

Each contemporary study was coded by publication year, context or scale, principal quantitative method, sustainability dimension, and operational contribution. Coding connects each formula to empirical practice and supports the capability rubric applied specifically in Section 6. The following

figure displays the five-stage pathway from scope definition and evidence identification to quality screening, benchmarking, and triangulation.

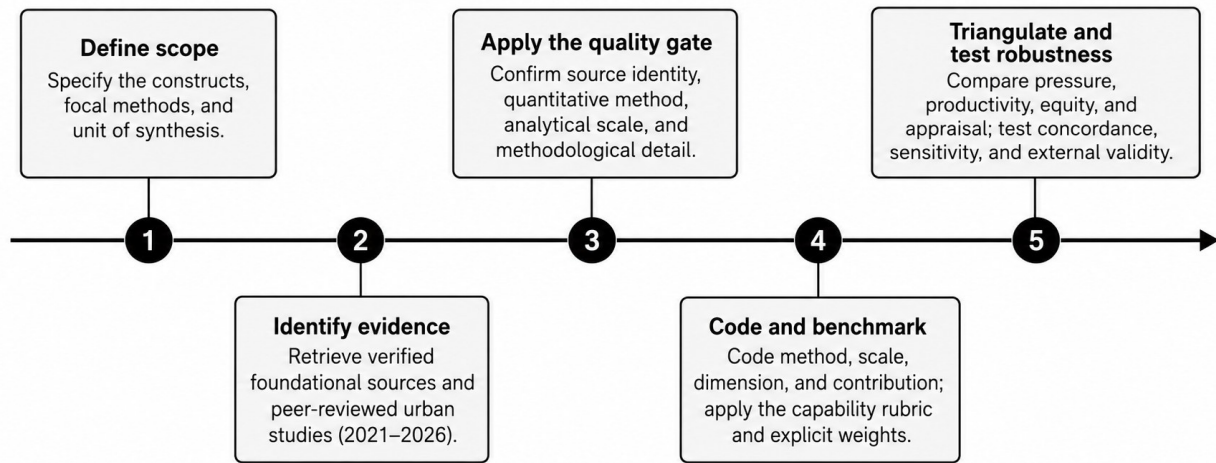


Figure 2. Structured method-centered review and quantitative synthesis workflow.

Table 1 then documents how the 25 contemporary studies contribute to the methodological synthesis. Bibliographic metadata and DOI resolution were checked against publisher or authoritative bibliographic records; the verification concerns source identity and relevance, not a claim that heterogeneous findings are directly poolable. The distribution of the corpus is itself informative: eleven studies operationalize efficiency or productivity methods, six address composite index construction, four address equity or land-use structure, and the remainder address footprint accounting, adjusted monetary accounting, project appraisal, and framework review. No single method dominates the recent literature, which is the empirical basis for the triangulation argument developed in Section 2.1.

Table 1: Contemporary evidence corpus and methodological relevance.

Study	Principal method	Primary dimension	Relevance to this review
Long (2021)	Super-efficiency SBM-DEA with undesirable outputs	Eco-efficiency	Demonstrates frontier efficiency with environmental burdens.
Liu, Yang, et al. (2022)	EFA/CFA metric-system validation	Composite sustainability	Formalizes and validates multidimensional urban sustainability metrics.
Kovács et al. (2022)	Ecological footprint and biocapacity	Ecological carrying capacity	Shows metropolitan ecological overshoot and scale sensitivity.
Liu, Qu, et al. (2022)	Emergy-EF + DEA-SBM	Ecological security / efficiency	Integrates footprint and frontier-efficiency logics.
Yao et al. (2022)	GML + coupling coordination + spatial analysis	Low-carbon urban economy	Adds temporal productivity and spatial evolution.
Wang et al. (2022)	GTFP + difference-in-differences	Smart-city productivity	Links policy intervention to green productivity.
Li & Cheng (2022)	Green TFP productivity indicator	City-size productivity	Tests city-size advantages under environmental constraints.
Zou et al. (2024)	Super-SBM + network / spatial methods	Urban eco-efficiency	Combines efficiency, network evolution, and influencing factors.
Wu & Bai (2022)	Ecological footprint	Ecological pressure	Tracks spatial and temporal footprint change.
Iannillo & Fasolino (2021)	Land-use mix indicators	Urban form	Clarifies measurable functional and social mix indicators.
Wu & Kim (2021)	Gini + longitudinal analysis	Green-space equity	Separates quantity from equality of green provision.
Guan et al. (2023)	Bayesian quantile regression	Green-space equity	Shows equity effects vary across the equity distribution.
Lu et al. (2024)	Geospatial access-equity assessment	Green-space justice	Demonstrates multi-type spatial-access inequities.
Khodakarami et al. (2023)	Spatial modeling + MCDA	Neighborhood sustainability	Integrates ecosystem services, hazards, and urban structure.

Malah & Bahi (2022)	PCA + GIS / remote sensing	Urban sustainability index	Uses PCA to reduce indicator dimensionality.
Foroozesh et al. (2022)	Fuzzy BWM + AHP + TOPSIS-GIS	Urban development sustainability	Provides multi-weighting and ranking architecture.
Wilbers et al. (2022)	Cost-benefit analysis	Blue-green infrastructure	Demonstrates economic appraisal of green infrastructure.
Medeiros et al. (2024)	Large-scale trip-data analysis of BRT, taxi, and TNC integration	Mobility integration	Updates the Curitiba case beyond its historical reputation and specifies first- and last-mile evidence.
Zhu et al. (2023)	Green GDP + BP neural network	Environmental-economic accounting	Extends GDP correction for resource and pollution costs.
Jiang et al. (2022)	Spatial econometric models + GTFP	Urban sprawl and productivity	Demonstrates spatial effects on green productivity.
Zhao et al. (2022)	GTFP + panel / spatial analysis	Green innovation	Links innovation to productivity and spatial spillovers.
Furlan et al. (2025)	DEA + neural networks + ANOVA	Composite sustainability	Integrates index construction, maturity classes, and longitudinal targets.
Wang & Jiang (2025)	Super-SBM + configuration analysis	Urban eco-efficiency	Updates undesirable-output efficiency with institutional and public-concern factors.
Saraswat et al. (2025)	Systematic review of 95 studies	Assessment frameworks	Updates methods, data-driven assessment, governance, and standardization gaps.
Marjanović et al. (2026)	Benefit-of-the-Doubt DEA composite index	Smart-city sustainability	Demonstrates perception-based, endogenous weighting across six dimensions.

3.3 Quantitative Method-Benchmarking Strategy

The eight established methods are appraised on a 1–5 capability rubric across ecological coverage, economic coverage, equity coverage, temporal adaptability, spatial scalability, data feasibility, uncertainty transparency, and policy interpretability. The anchors are: 1 = not directly represented; 2 = indirect or weak proxy; 3 = feasible only with material augmentation; 4 = direct but context-dependent capability; and 5 = direct, central, and well-established capability. The complete rating matrix is reported in Section 6.1, and the limitations discussion in Section 7 treats the ratings as author judgments requiring future expert validation rather than as observed city outcomes.

Robustness is examined under four fully disclosed weighting vectors: balanced, ecology-priority, economy-priority, and equity-priority. The balanced vector assigns .125 to each of the eight rubric dimensions. Each priority vector assigns .40 to its focal coverage dimension (ecological, economic, or equity coverage), .10 to each of the two remaining coverage dimensions, and .08 to each of the five procedural dimensions—temporal adaptability, spatial scalability, data feasibility, uncertainty transparency, and policy interpretability. All vectors add up to 1.00. The weighted capability scores determine the rankings, and tied scores receive the mean of the ranks they occupy. The degree of agreement between the four rankings is quantified by Kendall’s coefficient of concordance adjusted for tied ranks, while Spearman rank-order correlations assess the agreement between each priority ranking and the balanced ranking. These statistics are descriptive because the inputs are appraisal ratings rather than observed data.

4. Quantitative Methods and Formulas

In this section, eight existing quantitative approaches are contrasted with the proposed approach, the SUEI. Equations 1–8 formalize the existing approaches, while Equations 9–12 show how benchmark normalization, pillar aggregation, and final geometric aggregation are calculated for the proposed index. The formulas are explained in terms of their scales, data demands, and restrictions, after which each subsection ends with a connection between the formula and the current research cited in Table 1.

4.1 Ecological Footprint (EF)

The ecological footprint converts a city’s resource consumption and waste generation into the biologically productive land and sea area required to supply and absorb it, expressed in global hectares (gha).

$$EF = \sum_i (C_i / Y_i) \times EQF_i \quad (1)$$

where C_i = consumption of product or category i (tonnes); Y_i = world-average yield for i (t/ha); and EQF_i = the equivalence factor converting land type i to standardized global hectares.

Comparing EF to a city-region's biocapacity (BC, in gha) yields the ecological deficit or reserve, $BC - EF$; a negative value indicates that the urban economy is running a biocapacity deficit financed by imports and by drawdown of natural capital elsewhere. The principal limitation is that EF is a static, backward-looking flow measure: it cannot capture stock depletion of non-renewable resources or qualitative ecosystem degradation, which is why Section 2.1 pairs it with efficiency and equity evidence rather than treating it as a summary of sustainability.

Recent urban applications confirm both the usefulness and the limitations of the EF framework. Kovács et al. (2022) combined ecological footprint and biocapacity accounting to demonstrate substantial metropolitan ecological overshoot in Budapest and its agglomeration, while Wu and Bai (2022) tracked spatiotemporal footprint changes across resource-based cities. These studies reinforce two principles for urban-economy analysis: EF should be normalized per capita as well as reported in absolute terms, and territorial results should be interpreted alongside trade dependence, urbanization, and biocapacity context. Integrated work combining improved ecological-footprint accounting with DEA-SBM further demonstrates that footprint pressure and resource-use efficiency are complementary rather than substitutable constructs (Liu, Qu, et al., 2022).

4.2 Composite Sustainability Index (CSI)

Composite indices combine directionally normalized indicators for multidimensional comparison. To prevent an undesirable indicator such as emissions from increasing measured sustainability, the aggregation uses a direction-corrected normalized value z_{ij} :

$$CSI_i = \sum_j w_j z_{ij}, \text{ with } \sum_j w_j = 1 \quad (2)$$

where z_{ij} = the normalized score for city i on indicator j , calculated as a benefit or cost indicator using the directional rules in Equations 9 and 10; and w_j = the transparent indicator weight.

Direction correction has a concrete consequence: higher green-space provision increases CSI, whereas higher emissions or inequality decrease it. Without this step, the index rewards exactly the outcomes it is intended to penalize, which is the most common construction error in applied composite indexing. CSI remains sensitive to normalization bounds and weights. Equal weighting, PCA, entropy, AHP, BWM, and benefit-of-the-doubt approaches embody different assumptions and may reorder cities; consequently, the weighting rationale, multicollinearity diagnostics, and rank sensitivity must all be reported. Section 6.2 demonstrates this sensitivity quantitatively for the capability rubric used in this review.

Recent studies exemplify increasingly explicit index construction. Liu, Yang, et al. (2022) validate a multidimensional metric system through exploratory and confirmatory factor analysis; Malah and Bahi (2022) reduce indicator dimensionality with PCA; Foroozesh et al. (2022) integrate BWM, AHP, TOPSIS, fuzzy logic, and GIS; and Furlan et al. (2025) combine DEA, neural networks, and inferential validation. Marjanović et al. (2026) further apply benefit-of-the-doubt DEA to allow endogenous weighting of perception-based smart-city dimensions. Together, the above-mentioned studies support reporting indicator direction, bounds, weights, redundancy tests, and sensitivity rather than only a final index value.

4.3 Location Quotient (LQ) for Economic Specialization

The location quotient measures whether a city is relatively specialized in a given (often green or knowledge-intensive) economic sector compared with a reference region, indicating potential agglomeration advantages for a sustainable-economy transition.

$$LQ = (e_{i,c} / e_{t,c}) / (e_{i,r} / e_{t,r}) \quad (3)$$

where $e_{i,c}$ = employment in sector i in city c ; $e_{t,c}$ = total employment in city c ; and $e_{i,r} / e_{t,r}$ = the corresponding ratios for the reference region, for example, the national economy.

An LQ above 1 indicates local specialization, but a sustainable-economy interpretation additionally requires a declared green-sector taxonomy, a consistent reference economy, and longitudinal employment or value-added data. Without these choices, LQ measures specialization but not environmental performance, which is why the capability rubric in Section 6.1 assigns it the lowest possible ecological-coverage score.

For sustainable urban-economy research, LQ is most informative when green-sector specialization is linked to productivity, innovation, or employment growth rather than interpreted as an outcome in itself. Recent evidence on green innovation and green total-factor productivity indicates that technological and sectoral change can generate productivity gains and spatial spillovers across cities (Zhao et al., 2022). LQ can therefore be used as an explanatory or stratification variable in panel models examining whether specialized green-industrial clusters translate into broader sustainable-economy performance.

4.4 Green (Adjusted) GDP

Green GDP subtracts the monetized cost of resource depletion and environmental degradation from conventional GDP, correcting the standard economic output measure for its ecological externalities.

$$GGDP = GDP - D_n - D_e \quad (4)$$

where GDP = conventional gross domestic product; D_n = the monetized value of natural-resource depletion; and D_e = the monetized cost of environmental or pollution damage.

Green GDP is conceptually powerful but empirically contested, because monetizing D_n and D_e requires shadow-pricing methods—contingent valuation, hedonic pricing, or damage-cost estimation—that vary widely in reliability and are rarely available at intra-urban resolution. This data dependence is the reason the rubric in Section 6.1 scores green GDP low on spatial scalability and data feasibility despite its high policy interpretability.

Zhu et al. (2023) illustrate a contemporary green-GDP accounting approach that explicitly incorporates environmental pollution and resource-consumption costs within an environmental-economic accounting framework. For urban applications, the principal methodological requirement is transparency: depletion prices, pollution-damage values, valuation years, geographic allocation rules, and uncertainty ranges should be reported so that adjusted output can be reproduced and compared over time.

4.5 Data Envelopment Analysis (DEA) for Urban Eco-Efficiency

DEA is a non-parametric linear-programming method that benchmarks the relative efficiency of decision-making units. Equation 5 gives the conventional constant-returns-to-scale multiplier model as a transparent baseline (Charnes et al., 1978):

$$\max h_k = \sum_r u_r y_{rk}, \text{ subject to } \sum_i v_i x_{ik} = 1, \sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0 \forall j, \text{ and} \quad (5) \\ u_r, v_i \geq \varepsilon > 0$$

where h_k = the efficiency of city k ; y_{rk} = desirable output r ; x_{ik} = input i ; u_r and v_i = non-negative output and input weights; and ε = a small positive lower bound preventing zero-weight exclusion.

The baseline model treats every output as desirable and every input as costly, so pollution can enter it only by the analytically inconsistent device of relabeling an emission as an input. Slack-based measure models with explicitly declared undesirable outputs avoid this by allowing desirable outputs to expand while undesirable outputs contract within the same efficiency evaluation; the radial baseline in Equation 5 cannot represent that asymmetry. Applications in which pollution is part of production should therefore use the SBM formulation and state which outputs were designated undesirable.

A second interpretive rule concerns bounds. In conventional CCR and BCC specifications, efficiency is bounded at or below 1, with 1 indicating the estimated frontier. Super-efficiency models intentionally permit values above 1 in order to discriminate among frontier units. Scores from conventional and super-efficiency specifications therefore require separate interpretation and should not be pooled without harmonization.

Long (2021), Wang and Jiang (2025), and Zou et al. (2024) apply Super-SBM logic to urban eco-efficiency, while Yao et al. (2022) use a Global Malmquist–Luenberger framework for temporal change. Green-productivity and policy studies further combine efficiency measures with panel, quasi-experimental, or spatial models (Li & Cheng, 2022; Wang et al., 2022; Zhao et al., 2022).

A future city-panel application should specify land, energy, labor, and capital as inputs; value added and well-being as desirable outputs; and CO₂, air pollution, or waste as undesirable outputs. The number of cities must substantially exceed the combined input–output dimension, and bootstrap, slack, and temporal robustness results should be reported.

4.6 Shannon Diversity Index for Land-Use Mix

The Shannon index measures both the richness and the evenness of land-use categories. For cross-city comparison, it should be normalized by its theoretical maximum for k categories:

$$H' = -\sum_i p_i \ln(p_i) / \ln(k) \quad (6)$$

where p_i = the share of land-use category i ; k = the number of harmonized land-use categories; and H' ranges from 0 to 1 when $k > 1$.

H' approaches 0 under single-use dominance and 1 when the defined categories are evenly represented. Because the denominator $\ln(k)$ depends on the classification adopted, an unnormalized Shannon value computed over eight categories is not comparable with one computed over twelve; normalization is what makes the measure transferable between cities.

Iannillo and Fasolino (2021) show that land-use mix can be operationalized through multiple functional and social-mix indicators rather than treated as an intuitive design quality. In a sustainable urban-economy framework, the Shannon index should therefore be computed at a clearly defined spatial grain and paired with accessibility, travel, or economic indicators. Cross-city inference remains sensitive to zoning taxonomy and spatial grain, so identical categories and comparable analytical units are a precondition for comparison rather than a refinement of it.

4.7 Gini Coefficient for Spatial and Environmental Equity

Spatial or environmental equity should be estimated with population weights whenever neighborhoods differ in population. The population-weighted Gini coefficient is:

$$G^w = [\sum_i \sum_j w_i w_j |x_i - x_j|] / [2\mu^w (\sum_i w_i)^2], \text{ where } \mu^w = \sum_i w_i x_i / \sum_i w_i \quad (7)$$

where x_i and x_j = amenity, access, or environmental-quality values in spatial units i and j ; w_i and w_j = their resident populations; and μ^w = the population-weighted mean. An unweighted form is appropriate only for units of equal population.

G^w ranges from 0, denoting an equal person-level distribution across the defined units, toward 1, denoting high inequality. It should be reported together with absolute provision, a Lorenz curve, and subgroup or neighborhood disaggregation, because equal scarcity produces a low Gini value while representing a poor outcome. Equity, in other words, constrains the interpretation of provision rather than substituting for it.

Recent studies demonstrate why equity should be treated as a separate pillar rather than inferred from aggregate provision. Wu and Kim (2021) used the Gini coefficient longitudinally across Chinese cities and showed that green-space quantity and equality do not necessarily move together. Guan et al. (2023) used Bayesian quantile regression to show that the relationship between green-space spatial pattern and equality varies across the equity distribution, while Lu et al. (2024) assessed access equity for multiple types of urban green space across 263 cities. These studies support reporting both absolute provision and distributional inequality, preferably with subgroup or neighborhood-level disaggregation.

4.8 Net Present Value (NPV) for Green Infrastructure Investment

For project-level economic appraisal of green or sustainable infrastructure—district heating, urban forests, or bus rapid transit (BRT) corridors—discounted cash-flow analysis determines whether lifetime co-benefits justify upfront capital cost.

$$NPV = \sum_t [(B_t - C_t) / (1 + r)^t] \quad (8)$$

where B_t = monetized benefits in year t , such as energy savings, avoided health costs, and property-value uplift; C_t = costs in year t ; r = the discount rate; and t = the year index over the project's appraisal horizon.

A positive NPV indicates that the sustainable-infrastructure investment is economically justified once externalities are internalized. Sensitivity to the chosen discount rate r is a standard robustness check, because environmental co-benefits accrue over long, multi-decade horizons and are therefore heavily affected by discounting.

The economic appraisal of blue-green infrastructure in Oslo by Wilbers et al. (2022) illustrates the importance of comparing multiple intervention strategies and monetized benefit streams. For rigorous urban-economy analysis, NPV should therefore be accompanied by discount-rate sensitivity, alternative benefit valuations, time-horizon scenarios, and, where uncertainty is material, probabilistic simulation. A single positive NPV should not be interpreted as robust if modest changes in the discount rate or in externality prices reverse the sign of the result. Because NPV answers a question about a specified intervention rather than about a city, it is excluded from the SUEI pillars defined in Section 4.9.

4.9 Proposed Sustainable Urban Economy Integrated Index (SUEI)

SUEI begins with directionally consistent normalization against fixed policy, scientific, or historically stable bounds L_j and U_j rather than the observed minimum and maximum of a changing city sample. Fixed bounds matter because sample-relative normalization makes a city's score depend on which other cities happen to be included, so that adding or removing an observation changes results without any underlying change in performance. Scores are clipped to the interval $[0, 1]$. For a beneficial indicator:

$$z_{ij} = \text{clip}[(x_{ij} - L_j) / (U_j - L_j), 0, 1] \quad (9)$$

For an undesirable indicator such as emissions or inequality, the direction is reversed:

$$z_{ij} = \text{clip}[(U_j - x_{ij}) / (U_j - L_j), 0, 1] \quad (10)$$

Pillar scores are then calculated as weighted sums of the normalized indicators assigned to each pillar:

$$E_i = \sum_{j \in E} \alpha_j z_{ij}, \quad P_i = \sum_{j \in P} \beta_j z_{ij}, \quad S_i = \sum_{j \in S} \gamma_j z_{ij}, \quad \text{with } \sum \alpha_j = \sum \beta_j = \sum \gamma_j = 1 \quad (11)$$

where set E contains physical-pressure and ecological capacity indicators; set P contains adjusted output, productive efficiency, and green specialization; and set S contains access, distribution, and normalized land-use diversity. Project-level NPV is reported separately and is not treated as a citywide pillar indicator. The three pillar scores are then combined geometrically:

$$\text{SUEI}_i = (E_i \times P_i \times S_i)^{(1/3)} \quad (12)$$

The geometric mean in Equation 12 is partially compensatory: it penalizes imbalance more strongly than an arithmetic mean but does not eliminate trade-offs. A city scoring 0.8, 0.8, and 0.2 across the three pillars obtains an arithmetic mean of 0.60 but a SUEI of 0.50, so severe weakness in one pillar cannot be fully offset by strength in the others. Since the product term approaches zero as any pillar approaches zero, the index also communicates that a pillar score of zero is not merely a poor result but a disqualifying one. To avert double counting, each raw indicator is assigned to only one of the three pillar sets in Equation 11; a derived index is not combined with its own components; and pairs with $|r| \geq .80$ or a variance inflation factor (VIF) ≥ 5 are reduced through theory-led selection or PCA before aggregation. Pillar results, the SUEI value, benchmark sensitivity, and weight sensitivity must all be reported together, since the index is designed to make the effect of these choices visible rather than to conceal it.

4.10 Cross-Model Validation and Robustness Tests

Empirical testing should include comparison of city rankings using Spearman's rho, Kendall's tau, and Kendall's W; identification of redundancy through VIF and PCA; and measurement of the stability of the ranks using Monte Carlo resampling of permissible weights and benchmarking bounds. Panel data analysis could employ the GML or Malmquist indexes to separate the efficiency changes from technological changes (Yao et al., 2022), whereas spatial econometric models could be used for identifying dependence and spill-overs (Jiang et al., 2022; Zhao et al., 2022). External validity would involve testing against independent outcomes that were not used to construct SUEI.

Table 2 brings together what each approach is supposed to output, the basic information that it needs to have, which robustness tests should be applied to it, and current evidence supporting it, thus bridging Equations 1 through 12 with reproducible empirical applications. Reading across a row shows what a study must report to be reproducible; reading down the robustness column shows that every approach requires a sensitivity test and that only the specific parameters differ.

Table 2: Operationalization of the eight established methods and the proposed SUEI.

Method	Typical output	Core data	Recommended robustness check	Recent support
EF (Eq. 1)	gha per capita; total gha	Resource consumption, emissions, biocapacity	EF/BC ratio; sensitivity to consumption boundary	Kovács et al., 2022; Wu & Bai, 2022
CSI (Eq. 2)	0–1 or 0–100	Normalized multidimensional indicators	PCA and VIF; alternative weights; rank sensitivity	Liu, Yang, et al., 2022; Malah & Bahi, 2022
LQ (Eq. 3)	Ratio	Sector employment or output	Longitudinal LQ; link to productivity and innovation	Zhao et al., 2022
Green GDP (Eq. 4)	Currency	GDP, depletion, environmental damage	Shadow-price scenarios; uncertainty bands	Zhu et al., 2023
DEA / Super-SBM (Eq. 5)	Efficiency score	Inputs, desirable outputs, undesirable outputs	Slacks; bootstrap; GML change; returns-to-scale test	Long, 2021; Zou et al., 2024
Shannon H' (Eq. 6)	Normalized entropy index (0–1)	Land-use category shares	Scale and category-count sensitivity	Iannillo & Fasolino, 2021
Weighted Gini (Eq. 7)	0–1	Amenity or access values by spatial unit, with populations	Lorenz curve; subgroup analysis; weighted vs. unweighted	Wu & Kim, 2021; Guan et al., 2023; Lu et al., 2024
NPV (Eq. 8)	Currency	Discounted costs and monetized benefits	Discount-rate and benefit-value sensitivity	Wilbers et al., 2022
SUEI, proposed (Eqs. 9–12)	0–1	Fixed-bound ecological, economic, and social pillar indicators	Direction and bound tests; VIF and PCA; alternative weights; Monte Carlo ranks	This review

Note: Robustness procedures indicate minimum reporting requirements for future empirical applications; NPV remains a project-level appraisal tool rather than a city outcome score. gha = global hectares. Source: Authors' elaboration from the sources listed in Table 1.

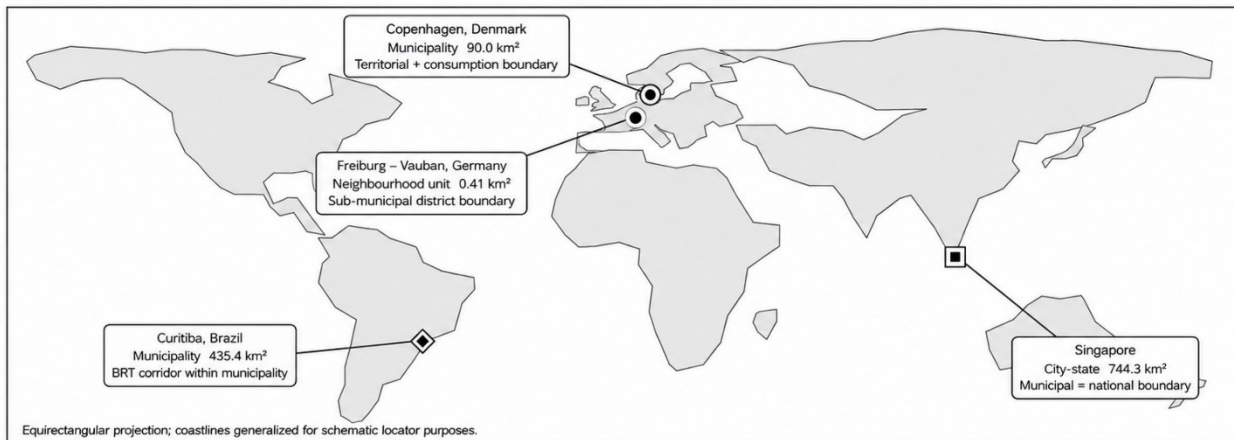
5. Illustrative Case Applications

The four applications below are interpretive vignettes selected for variation in scale, policy domain, density, and trade dependence. They show how the reviewed methods could be applied and where their interpretations may diverge. No new EF, DEA, Gini, green GDP, Shannon, or NPV values are calculated; consequently, the cases are not used as comparable empirical observations in the capability analysis of Section 6.

The descriptive statistics reported in this section are official administrative and census figures used solely to establish the scale and boundary of each analytical unit. They are contextual descriptors, not sustainability outcomes, and they are drawn from different national statistical systems using different

reference dates and definitions—which is itself part of the argument this section makes. Figure 3 locates the four cases geographically and places their analytical units on a common logarithmic area scale.

A Geographic position, analytical unit, and accounting boundary of the four illustrative cases



B Analytical grain of each case on a common logarithmic area scale

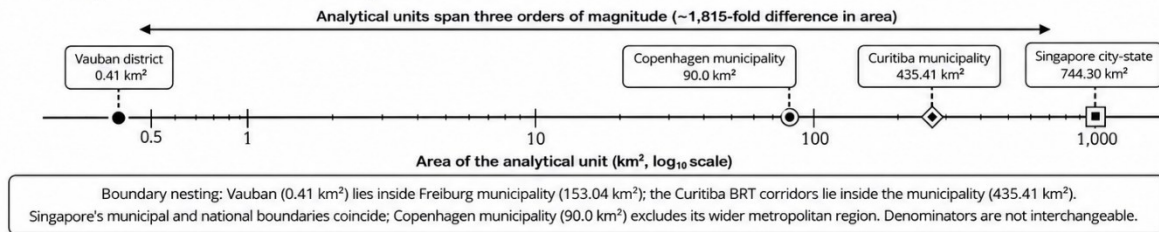


Figure 3. Geographic position, analytical unit, and accounting boundary of the four illustrative cases.

Figure 3 makes the central methodological point of this section visible. The analytical units span roughly three orders of magnitude, from 0.41 km² for the Vauban district to 744.3 km² for Singapore—an approximately 1,815-fold difference in area. Because the four units are also nested differently within their administrative hierarchies, a per-capita or per-hectare indicator computed for one of them has no automatic equivalent in another. Vauban lies inside a municipality that is 373 times its size; the Curitiba BRT corridors lie inside a municipality that extends well beyond them; Singapore's municipal and national boundaries coincide, so no larger domestic denominator exists; and Copenhagen municipality excludes the metropolitan region within which much of its commuting and consumption occurs. These are not incidental descriptive facts but the reason that Sections 4.1, 4.6, and 4.7 each require an explicitly declared spatial unit before their formulas can be applied comparatively.

5.1 Freiburg-Vauban District, Germany

Freiburg and its Vauban district are established examples of coordinated land-use, transport, and environmental planning (Beatley, 2000; Buehler & Pucher, 2011). Vauban occupies 41 hectares and houses approximately 5,500 residents in 2,472 households, with 172 registered cars per 1,000 residents (City of Freiburg, 2024). Its neighborhood scale makes it useful for illustrating how land-use categories, household energy demand, and travel behavior could be linked within a multi-metric assessment.

Methodological relevance: normalized Shannon diversity (Equation 6) can describe land-use evenness, while ecological-footprint or energy indicators (Equation 1) can represent consumption pressure. These measures would require harmonized parcel categories, a stated district boundary, and primary consumption data; this review does not infer numerical performance from descriptive planning features, and the figures above are reported only to fix the analytical unit.

The principal lesson is one of scale. At the district grain, Vauban records approximately 13,415 residents per square kilometer, whereas Freiburg municipality as a whole records approximately 1,521

residents per square kilometer (City of Freiburg, 2021, 2024)—a difference of about 8.8 times produced entirely by the choice of analytical unit rather than by any difference in the underlying city. Neighborhood design evidence therefore cannot automatically be generalized to the municipal economy, and a city-panel study should report both district results and the citywide denominator against which they are interpreted.

5.2 Curitiba, Brazil: Bus Rapid Transit and Integrated Land Use

Curitiba's BRT system and land-use integration provide an illustration of how mobility outputs, operating inputs, accessibility, and investment costs may be evaluated over time. The municipality covers 435.41 km² and recorded 1,773,718 residents at the 2022 census, giving a demographic density of approximately 4,079 residents per square kilometer (IBGE, 2023). Medeiros et al. (2024) show that contemporary analysis must also consider the integration of taxis and transport-network companies with the BRT system rather than freezing the case at its historical reputation.

Methodological relevance: Curitiba could enter a DEA model (Equation 5, or its SBM extension) only within a common multi-city dataset using identical inputs and outputs; without such a dataset it cannot validly be described as lying on an efficiency frontier. Corridor investments may also be evaluated through NPV (Equation 8), provided that capital, operating, accessibility, health, and congestion assumptions are disclosed.

The case therefore illustrates temporal stability and boundary definition. The BRT corridors form a linear subset of a municipality of 435.41 km², so corridor-level ridership or accessibility results describe the corridor rather than the city, and a historically successful transport intervention requires longitudinal assessment of ridership, service quality, first- and last-mile integration, costs, and distributional accessibility.

5.3 Singapore: Land-Constrained High-Density Sustainability

Singapore illustrates the interpretive challenge faced by compact, highly trade-dependent city-states. Its land area of 744.3 km² (Department of Statistics Singapore, 2025) supports a population of approximately 6.11 million, or roughly 8,209 residents per square kilometer. A consumption-based ecological footprint is designed to include resource demand embodied in imports and therefore exposes reliance on external biocapacity rather than understating it (Wackernagel & Rees, 1996).

Methodological relevance: EF should be interpreted together with land-use efficiency, accessibility, and population-weighted equity (Equation 7). High efficiency within a small territory does not cancel external ecological demand, while a large consumption footprint does not by itself describe internal accessibility or distribution.

Singapore consequently demonstrates why accounting boundary and construct validity matter. Because its municipal and national boundaries coincide, there is no larger domestic reference region against which a location quotient could be computed in the conventional way, and territorial emissions, consumption-based footprint, density efficiency, and distributive access answer different questions that should remain visible before integration.

5.4 Copenhagen, Denmark: Carbon Accounting and Infrastructure Appraisal

Copenhagen is retained as an illustration of combining physical decarbonization evidence with the appraisal of long-lived transport, heating, and green-infrastructure investments. The municipality contained 667,124 residents on 1 January 2025 (City of Copenhagen, 2025) within an area of approximately 90 km², giving a density of roughly 7,412 residents per square kilometer. The relevant analytical distinction in this case is between emissions decoupling and adjusted economic accounting.

Methodological relevance: comparing GDP growth with territorial or consumption emissions measures relative or absolute decoupling; it is not, by itself, a green-GDP calculation. Green GDP additionally requires explicit monetary estimates of depletion and environmental damage (in Equation 4). Project NPV (in Equation 8) addresses a different question again: whether a specified intervention's discounted benefits exceed its costs.

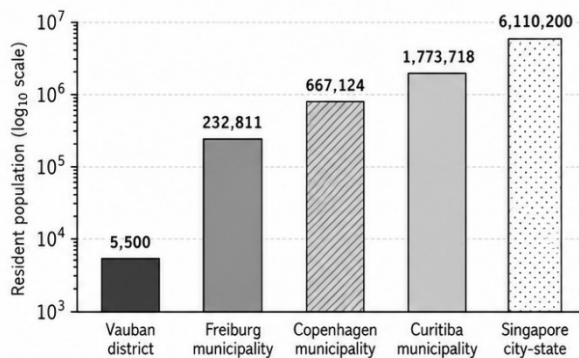
The case supports transparent reporting of carbon boundaries, valuation years, discount rates, appraisal horizons, and health or congestion co-benefits. Because the municipal boundary excludes much of the

surrounding metropolitan region, consumption and commuting emissions attributed to the municipality depend directly on where that line is drawn. These requirements are consistent with the scenario-based urban green infrastructure appraisal demonstrated by Wilbers et al. (2022).

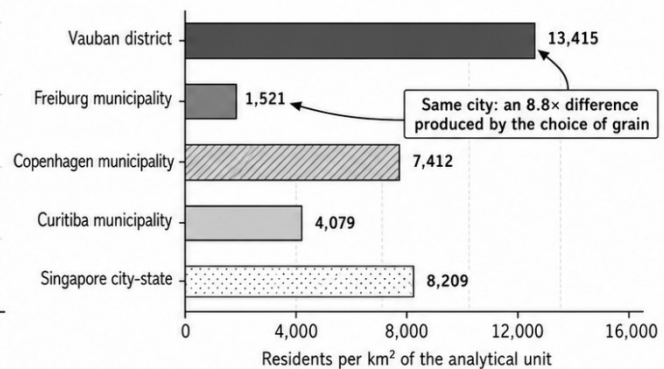
5.5 Comparative Reading of the Four Cases

Figure 4 assembles the descriptive context of the four cases and the methods each can illustrate. Figure 4.a and Figure 4.b show the resident population and population density of each analytical unit; Figure 4.c isolates the scale effect within a single city; and Figure 4.d maps each case onto the eight established methods.

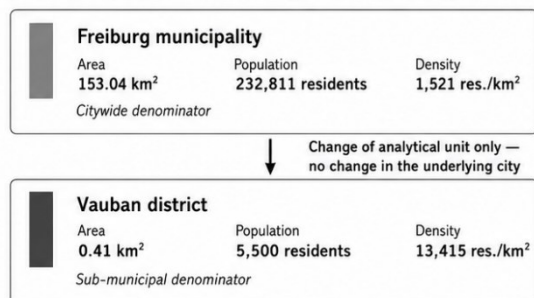
(a) Resident population of each analytical unit



(b) Population density at the reported grain



(c) Scale dependence of a single case: Freiburg



Vauban additionally reports 2,472 households and 172 cars per 1,000 residents, indicators that have no defined citywide equivalent at the same grain.

(d) Methods each case can illustrate

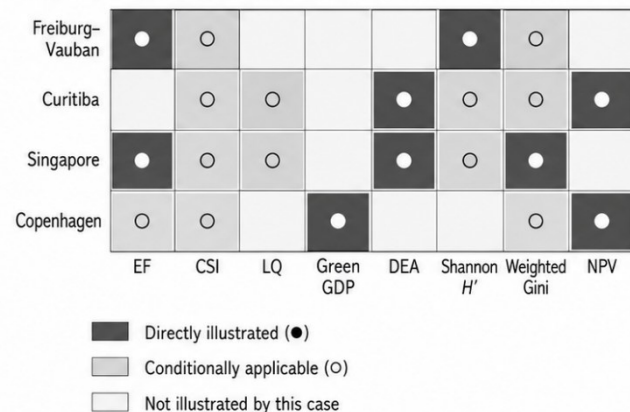


Figure 4. Comparative descriptive context and method coverage of the four illustrative cases.

Three patterns follow from the figure. First, population size and density are not aligned: Singapore has the largest population of the four units but a lower density than the Vauban district, so neither variable alone identifies the analytical grain in use. Second, Figure 4.c shows that Freiburg yields two defensible and very different density values depending only on whether the district or the municipal denominator is adopted, which is a direct illustration of the modifiable areal unit problem that Section 7 identifies as a threat to validity. Third, Figure 4.d shows that no case illustrates more than four methods directly, confirming that the cases are complementary demonstrations rather than repeated tests of the same construct. Table 3 summarizes the analytical function illustrated by each case and the principal caution governing its interpretation. The entries are conceptual mappings supported by the cited literature, not newly calculated city scores.

Table 3: Illustrative case-to-method mapping and interpretation limits.

Case	Primary metric(s)	Demonstrated strength	Key limitation exposed
Freiburg (Vauban)	Normalized Shannon H'; EF and energy indicators	Neighborhood-scale integration of form, mobility, and energy	District evidence is not automatically citywide; density varies about 8.8-fold with the denominator
Curitiba	DEA candidate; project NPV	Links mobility outputs, inputs, access, and investment	Frontier claims require a common multi-city dataset; corridors are a subset of the municipality
Singapore	Consumption EF; DEA; population-weighted Gini	Reveals external footprint, internal efficiency, and access	Imports are included in consumption EF; municipal and national boundaries coincide, so no domestic reference region exists
Copenhagen	Emissions decoupling; project NPV	Combines physical change with infrastructure appraisal	Decoupling is not equivalent to green GDP; the municipal boundary excludes the metropolitan region

6. Comparative Quantitative Synthesis

Section 5 established what the four cases can and cannot demonstrate. This section turns from case-based interpretation to the systematic comparison of the eight established methods, first through the capability rubric defined in Section 3.3 and then through the four weighting scenarios that test how far the resulting ordering depends on normative priorities.

6.1 Cross-Method Capability Matrix

Table 4 reports the disclosed rubric in full. The ratings compare analytical functions, not city outcomes or universal superiority. As defined in Section 3.3, a score of 5 indicates that a dimension is central and directly operationalized by the method; 4 indicates direct but context-dependent capability; 3 requires material augmentation; 2 is indirect; and 1 indicates that the dimension is not directly represented. Means are computed across the eight dimensions with equal weights and reported using conventional half-up rounding.

Under equal weighting, CSI and the Gini coefficient tie for the highest capability mean (4.13 out of 5), followed by LQ (3.88), DEA (3.75), Shannon diversity (3.63), NPV (3.50), green GDP (3.25), and EF (3.13). These means describe breadth of coverage across the eight rubric dimensions, not accuracy or validity: EF's lower mean reflects deliberately narrow coverage—it scores the maximum on ecological coverage and the minimum on equity coverage—rather than weak ecological validity. A method that measures one construct precisely will always score lower on a breadth rubric than a method designed to span several constructs, which is why Section 6.2 tests what happens when breadth is no longer weighted equally.

Table 4: Author-developed methodological capability scoring matrix.

Method	Ecolog y	Econom y	Equity	Tempor al	Spatia l	Data feasibilit y	Uncertaint y	Policy use	Mean
Ecological Footprint	5	2	1	3	4	3	3	4	3.13
Composite Sustainability Index	5	5	5	4	4	3	2	5	4.13
Location Quotient	1	5	2	5	5	5	4	4	3.88
Green GDP	4	5	2	4	2	2	2	5	3.25
DEA / Super-SBM	4	5	2	4	4	3	3	5	3.75
Shannon Diversity	3	2	2	4	5	5	4	4	3.63
Gini Coefficient	1	2	5	5	5	5	5	5	4.13
NPV	3	5	2	5	3	3	2	5	3.50

Figure 5 reproduces the same values visually, making both cross-method strengths and dimension-specific gaps comparable at a glance. The visual pattern shows two distinct profiles: CSI and, to a lesser extent, DEA are broad but weaker on uncertainty transparency, whereas Gini, LQ, and Shannon diversity are narrow in coverage but strong on the procedural dimensions of feasibility, scalability, and transparency.

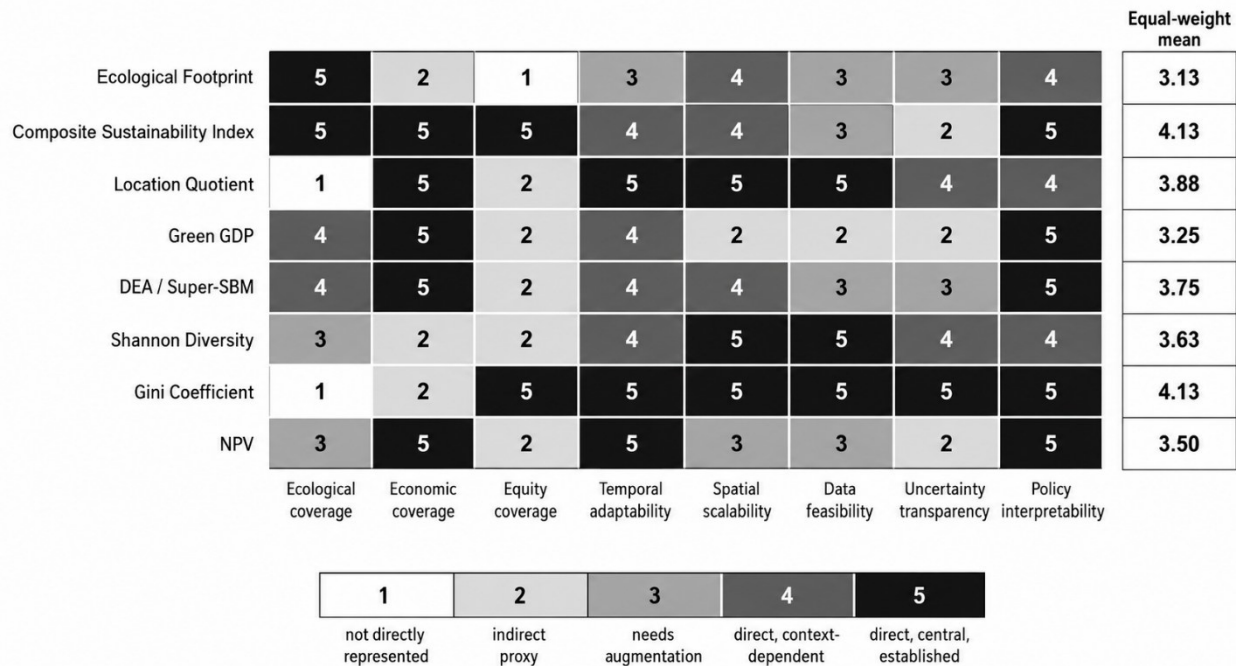


Figure 5. Methodological capability heatmap for the eight established methods.

6.2 Weighting Sensitivity and Rank Concordance

The four weighting vectors specified in Section 3.3 are applied to the capability scores in Table 4. Under the balanced vector each of the eight dimensions receives .125. Under each priority vector the focal coverage dimension receives .40, the two remaining coverage dimensions receive .10 each, and the five procedural dimensions receive .08 each. The resulting weighted scores generate the ranks reported in Table 5, in which lower numbers indicate greater capability under the stated priority and tied scores receive averaged ranks.

The pattern of movement is systematic rather than arbitrary. CSI ranks first in every scenario, because it is the only method rated 5 on all three coverage dimensions and is therefore insensitive to which of them is emphasized. The specialized methods move sharply in the direction of their specialization: EF rises from eighth under balanced weighting to third under ecological priority; LQ rises from third to second under economic priority but falls to eighth under ecological priority; and the Gini coefficient moves from a balanced tie for first to second under equity priority while falling to seventh under ecological priority. Green GDP and NPV occupy intermediate positions that improve modestly whenever monetary or economic criteria are emphasized.

Table 5: Method ranks under explicit alternative weighting priorities.

Method	Balanced	Ecology-priority	Economy-priority	Equity-priority
Ecological Footprint	8	3	8	8
Composite Sustainability Index	1.5	1	1	1
Location Quotient	3	8	2	3
Green GDP	7	4	5	7
DEA / Super-SBM	4	2	3	4
Shannon Diversity	5	5	7	5
Gini Coefficient	1.5	7	6	2
NPV	6	6	4	6

Note: Weight vectors are specified in Section 3.3 and restated in Section 6.2; every vector sums to 1.00. Lower rank numbers indicate greater capability under the stated priority, and tied scores receive averaged ranks.

Concordance across the four rankings is moderate. Kendall's coefficient of concordance, corrected for tied ranks, is $W = 0.525$ across the four scenarios and eight methods. Spearman rank-order correlations with the balanced ranking are $-.120$ for the ecological-priority ranking, $.587$ for the economic-priority ranking, and $.994$ for the equity-priority ranking. The equity-priority ordering is therefore almost identical to the balanced ordering, whereas the ecological-priority ordering is effectively unrelated to it—the single most consequential finding of the sensitivity analysis, since it means that an apparently neutral equal-weight ranking is much closer to an equity-weighted judgment than to an ecologically weighted one.

Figure 6 presents the same values as a grayscale rank matrix so that stable and priority-sensitive methods can be compared directly. These descriptive results demonstrate sensitivity to normative priorities; because the inputs are author ratings rather than sampled observations, they do not constitute city-level empirical validation and no inferential test is attached to them.

	Balanced	Ecology priority	Economy priority	Equity priority
Composite Sustainability Index	1.5	1	1	1
Gini Coefficient	1.5	7	6	2
Location Quotient	3	8	2	3
DEA / Super-SBM	4	2	3	4
Shannon Diversity	5	5	7	5
NPV	6	6	4	6
Green GDP	7	4	5	7
Ecological Footprint	8	3	8	8

Figure 6. Rank sensitivity across alternative weighting priorities.

6.3 Implications for City-Level Statistical Comparison

A future empirical test should construct a harmonized city-by-indicator panel with common years, spatial boundaries, benchmark bounds, and source definitions. Ecological-pressure measures, green GDP, Super-SBM efficiency, green-sector LQ, normalized land-use diversity, and population-

weighted equity may be compared at the city level; NPV should remain attached to clearly specified projects rather than assigned as a generic city score. Agreement can be assessed through the rank statistics used in Section 6.2, while predictive validity should use independent outcomes and out-of-sample error metrics. The scale evidence in Figure 3 implies a further requirement: the panel must fix one spatial unit per indicator and report it, because otherwise the ranking will partly reflect the choice of denominator rather than the performance being measured.

7. Discussion

The central result of this synthesis is that the reviewed methods are complements rather than interchangeable substitutes, because they operationalize distinct constructs, boundaries, and decision scales. Each formula in Section 4 answers a different question—pressure relative to biocapacity, monetized depletion and damage, relative productive efficiency, sectoral specialization, land-use evenness, distribution, or the viability of a specified investment—and CSI and the proposed SUEI integrate selected dimensions only under conditions of correct directionality, stable bounds, explicit weights, and control for indicator overlap. The capability scores in Table 4, depicted graphically in Figure 5, further support this complementarity: the tied equal-weight means of CSI and the Gini coefficient indicate comparable breadth rather than the superiority of either method, and the low mean score of EF reflects its specialization rather than a lack of ecological validity. Table 5 and Figure 6 further confirm that rankings depend on the stated priorities, with moderate agreement across the four scenarios (Kendall's W , tie-corrected = 0.525), with the ecology-priority scenario deviating the most from the balanced ranking.

The findings also align with a broader methodological shift from isolated indicators toward spatially and temporally explicit combinations. Integrating ecological-footprint accounting with DEA-SBM distinguishes absolute environmental pressure from relative efficiency (Liu, Qu, et al., 2022), while panel, policy-evaluation, network, and spatial approaches move green productivity analysis beyond static league tables (Wang & Jiang, 2025; Wang et al., 2022; Zhao et al., 2022; Zou et al., 2024). Equity analyses further reveal that greening, on average, does not preclude inequality in accessibility between neighborhoods and different social strata (Guan et al., 2023; Lu et al., 2024; Wu & Kim, 2021). Composite assessment is thus enhanced by validated factor structures, alternative and endogenous weighting, and sensitivity analysis (Foroozesh et al., 2022; Furlan et al., 2025; Liu, Yang, et al., 2022; Marjanović et al., 2026). The four examples clarify that integration is valid only if each measure preserves its original meaning and divergences between measures convey trade-off information rather than being averaged out. The four examples further corroborate this point because Vauban emphasizes the importance of neighborhood-city scaling, Curitiba highlights the data needs for frontier claims, Singapore defines territory versus consumption boundaries, and Copenhagen separates carbon accounting from monetary evaluation.

The resulting research agenda identifies three linked threats to validity: duplication, instability across space and time, and insufficient external testing. EF, green GDP, DEA, and composite indices may reuse energy, land, or emissions variables, so each raw indicator should be assigned to one pillar and screened through correlation analysis, VIF, and PCA before aggregation; PCA-based urban assessment and multivariate metric validation already demonstrate the value of this step (Liu, Yang, et al., 2022; Malah & Bahi, 2022). Spatial analysis must state boundary definitions, population denominators, and modifiable areal unit assumptions, because citywide environmental accounting and neighborhood equity measures operate at different resolutions (Khodakarami et al., 2023); the roughly 8.8-fold density difference between the Vauban district and Freiburg municipality reported in Section 5.1 illustrates how large that effect can be within a single city. Temporal robustness requires harmonized panels, panel DEA or Super-SBM with undesirable outputs, and GML or Malmquist decomposition where repeated observations permit separation of efficiency and technological change. Future studies should additionally use Monte Carlo simulation to vary admissible weights and benchmark bounds, report median ranks with uncertainty intervals, test spatial dependence through Moran's I and

appropriate spatial regression models, and examine discount rates, carbon prices, appraisal horizons, and co-benefit valuation in NPV and green-GDP applications.

External validity should finally be evaluated out of sample against independently measured emissions, accessibility, health, life satisfaction, or green productivity, consistent with current calls for greater standardization and empirical validation (Furlan et al., 2025; Saraswat et al., 2025). These conclusions should be interpreted within the review's limitations. The evidence synthesis is focused and method-centered rather than an exhaustive bibliometric review; its quality gate verifies source identity, quantitative operationalization, scale, and methodological transparency but is not a design-specific risk-of-bias instrument. The capability matrix reflects the authors' own judgments, so the concordance and correlation statistics in Section 6.2 describe the internal structure of those judgments rather than an external empirical regularity. The four cases are illustrative rather than representative, and their descriptive statistics come from different national statistical systems with different reference dates, so they establish scale and boundary but support no cross-case performance comparison. The proposed city panel, Monte Carlo, spatial, and predictive analyses are protocols for future replication rather than completed empirical tests, and these boundaries preclude any claim that SUEI has already been validated as a universal city ranking instrument. Nevertheless, the framework offers a practical contribution by making value judgments and data dependencies visible. A stronger next stage would preregister indicator-selection rules, use Delphi or multi-rater scoring with inter-rater reliability, publish harmonized data and code, and compare results across alternative spatial units and policy scenarios. In terms of policy, the message is equally specific: no headline index should be released on its own. Uncertainty or rank intervals, raw measures, pillar scores, the aggregated score, and a record of all duplicates that were removed must be released as a package. Such a structure turns differences between approaches into useful information about ecological stress, economic inefficiencies, distributional consequences, and investment alternatives.

8. Conclusion

The present paper proposes a consistent quantitative framework for measuring a sustainable urban economy without committing the fallacy of considering ecological stress, productive efficiency, economic specialization, spatial equity, land-use pattern, and project feasibility as equivalent outcomes. Its first contribution is methodological clarification: it links eight established tools and the new SUEI to their constructs, measurement scales, formulas, and interpretive limits. Several operational rules follow. Undesirable indicators need to be directionally transformed before aggregating CSI; the inputs, desirable and undesirable outputs of DEA need to be determined in a proper model instead of renaming pollution as input; the Shannon index needs to be normalized if the number of categories is unequal; the Gini coefficient needs to include the weights of population; green GDP needs monetary values of depletion and damage; and the NPV cannot be detached from a specific intervention. SUEI adds an integration layer with fixed-benchmark normalization, one-pillar assignment of raw indicators, and geometric aggregation, but does not substitute for its ecological, economic, and social components.

Second, the comparative results establish a clear requirement: comprehensive assessment must be based on a portfolio of methods rather than a particular preferred method. A bare minimum would be to use one physical stress indicator, one efficiency or productivity indicator, and one indicator of distribution; the adjusted monetary assessment is to be used only if the research questions involve environmentally adjusted economic output or the economic feasibility of a particular investment. The capability approach makes the normative implications of the method selection obvious. Although CSI ranks highest in all four weighting scenarios, the environmental, economic, and equity considerations substantially alter the relative ranks of the specialized methods, resulting in only moderate concordance between the rankings (corrected Kendall's $W = 0.525$) and, in the ecology-priority case, in a ranking which is almost completely unrelated to the balanced one. This finding is not confirmatory but purely descriptive since the inputs are openly disclosed author ratings; however, it shows how sensitive a supposedly objective ranking of cities may be to its own policy priorities.

The methodology proposed in this review sets out a replicable process for empirical implementation. Researchers must first create a harmonized set of city-by-indicator panels using a common time period, fixed spatial boundaries, identical source specifications, and policy-meaningful benchmark ranges. Prior to integration, redundancy should be tested with correlation coefficients, VIF statistics and principal component analysis, and each indicator must appear only once in a single pillar. Missing data rules, population bases, prices used, carbon factors, discount rates, and appraisal horizons should be disclosed prior to any result calculations. Robustness testing should be conducted through an examination of a variety of normalization and weighting methods, in conjunction with Monte Carlo rank intervals, productivity decomposition of the panel, spatial dependence tests, and out-of-sample validation relative to outcomes not included in index construction. When structural differences exist across cities, it might be more meaningful to cluster cities or perform archetype-based comparisons rather than trying to fit all observations onto a single linear ranking system. Most critically, raw indicators, pillar scores, SUEI, and sensitivity results must be reported side by side.

Thus, the strength of the proposed framework does not come from generating a universal sustainability rating but rather from setting up an audit trail from research question to indicator, method, scale, aggregation formula, robustness test, and policy implications. The framework itself is conservative; it is built on the understanding that while a city can be efficient, it may still be ecologically dependent; while a city can be green overall, there may be unequal access to green areas; while a city economy can be specialized, it can be fragile; and while a city may benefit from a good project, it is not thereby sustainable. In future multi-city studies, it will be important to test the framework against consistent longitudinal data, independent scoring, open computing procedures, and external validation of outcomes. Only if these conditions are met can SUEI be properly evaluated as a synthesis tool rather than accepted as a self-evident construct. Until then, SUEI's best purpose will be to organize complementary evidence and disclose the decision-making involved in integrated assessments.

Acknowledgements

The authors would like to thank the editor and anonymous reviewers for their valuable comments and suggestions, which helped improve the quality of this manuscript.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The original contributions presented in the study are included in the article/supplementary material; further inquiries can be directed to the corresponding author.

Institutional Review Board Statement

Not applicable.

CRedit Author Statement

Seyed Alireza Athari: Conceptualization, Methodology, Formal analysis, Writing – original draft. Nehme Azoury: Data curation, Investigation, Writing – review & editing. Dervis Kirikkaleli: Supervision, Validation, Writing – review & editing. All authors have reviewed and approved the final version of the manuscript.

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How to cite this article? (APA Style)

Athari, S. A., Azoury, N., & Kirikkaleli, D. (2026). Quantifying the sustainable urban economy through integrated metrics, models, and sensitivity analysis. *Journal of Contemporary Urban Affairs*, 10(2), 296-318. <https://doi.org/10.25034/jjcua.2026.v10n2-1>